1.

The idea is to find the volume of a cross sectional slice of the solid, and then add up all such volumes. We want to use a very thin slice, of thickness dx or dy, since we are going to be taking the limit as the thickness of each slice goes to zero to come up with an integral that gives us the volume of the solid. So, all we need to do to get the volume of a slice is multiply the thickness by the surface area of the slice.

$$\text{Volume of Solid} = \int_a^b (\text{Area of Slice})\,(\text{Thickness})$$

Example

A solid has a base that is bounded by the curves $y = x^2$ and

Example

Set up an integral that will give the volume of the region generated by revolving the region R about the x-axis

Example

Set up an integral that will give the volume of the region generated by revolving the region R about the y-axis

3.

A special case of the slice and sum method: we rotate a region from the xy-plane around an axis and use slices that are PARALLEL to the axis of revolution. The pieces are cylindrical shells and we can find the volume of the resulting solid using the shell formula.

$$\text{Volume of Solid} = 2\int_a^b (\text{radius of cylinder})(\text{height of cylinder})(\text{thickness})$$

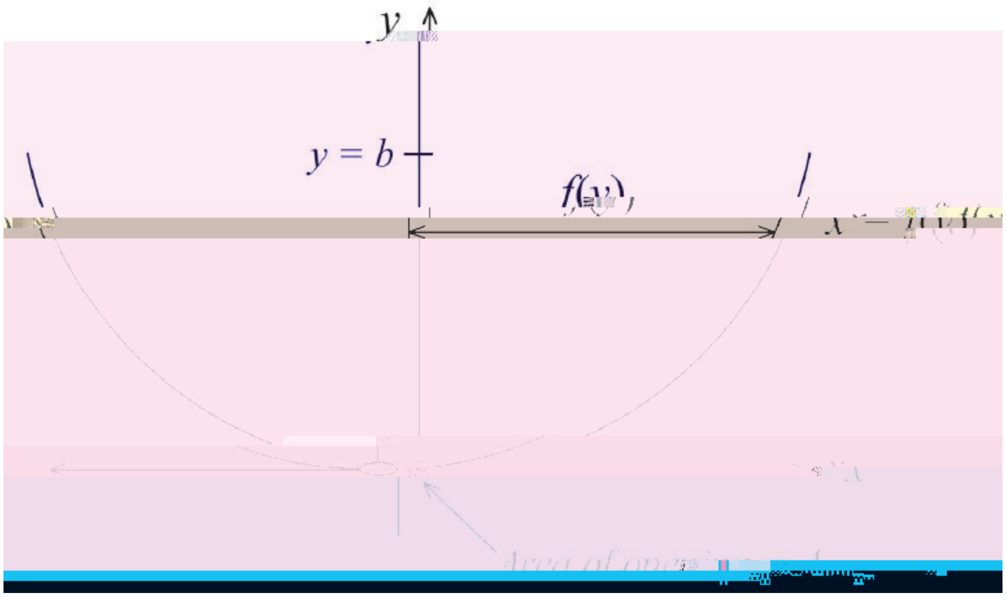
Example

Set up an integral that will give the volume of the region generated by revolving the region R about the y-axis.

Example

Set up an integral that will give the volume of the region generated by revolving the region R about the x-axis.

Before mechanical and digital clocks were invented, time was kept by water clocks (and other devices). A water clock consists of a large container with a hole in the bottom. When filled with water, the water drains through the hole in such a way that the surface of the water drops at a constant rate. In this way the clock is easily calibrated: the hours are marked on the container with equal spacing, and the distance between marks always corresponds to the same interval of time (like an hour).



The goal will be to find the formula for the function f that will define the shape of exactly such a water clock.

1. Assume that the water in the clock forms a solid of revolution generated by rotating the curve $x = f(y)$ between $y = 0$ and $y = b$ inclusive around the y-axis. Suppose that the water level is at a variable height y. Use the disk method to find an integral that will give the volume V of water in the container — use t as the dummy variable of integration.

2. As the water level in the clock drops, both the volume of water V and the water level y change over time. Use the Fundamental Theorem of Calculus to differentiate both sides of the above equation with respect to time t.

3. Explain why dy/dt is a constant. Since we are assuming dy/dt will be given (-1 inch per hour, for example), we need to think about dV/dt. … say that

5. Suppose you want to design a 24-hour clock that is $b = 1$ m high. The design is such that the water level drops 1/24 m each hour, so $k = 1/24$ m/h. Suppose the hole in the bottom of the tank has an area of $A = 0.02\ m^2$, so that the radius is approximately 8 cm or 3 in. Find and graph the function that describes the container, find the approximate radius of such a container, and find the volume of water needed to fill this container.